Proposal to observe the strong Van der Waals force in the low energy neutron-Pb scattering*

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Abstract

In the neutron-Pb scattering in the MeV. region, existence of the long range interaction has been known and people have hopefully expected to understand it as the effect of the electric polarization of the neutron. However the precise determination of α_n in the very low energy region indicates that the polarizability of the neutron α_n is around two order of magnitude smaller compared to what is expected in the phenomena in the MeV. region. We need another strong long range potential which decreases faster than r^{-4}. On the other hand, analysis of the low energy proton-proton data revealed the existence of the strong long range potential whose tail was $V(r) \sim -C/r^{\alpha}$ with $\alpha = 6.08$. The neuton-nucleus scattering is a good place to see such a force, since the strength C is magnified by factor A, the mass number. Using the long range parameters obtained in p-p, we predict the characteristic angular dependency of the n-Pb amplitude for fixed energy. Appearance of a cusp at $\nu = 0$, which points upward, in the once subtracted P-wave amplitude $a_1(\nu)/\nu$ is predicted.

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1 Introduction

By the neutron transmission experiments of Pb^{208} in the energy range of $0.05keV < T_{lab} < 40keV$, Schmiedmayer et.al.[1] successfully determined the electric polarizability of the neutron α_n. In the determination of α_n, they fitted the n-Pb^{208} cross section as

$$\sigma(\nu) = 11.508(5) + 0.69(9)\sqrt{\nu} - 448(3)\nu + 9500(400)\nu^2 \tag{1}$$

where σ and ν are measured in the units of barn and fm^{-2} respectively. The $\sqrt{\nu}$ term arises from the long range potential of the electric polarization of the neutron. From the coefficient of $\sqrt{\nu}$, they obtained the value $\alpha_n = (1.20 \pm 0.15 \pm 0.20) \times 10^{-3}\ fm^3$. On the other hand, when all the forces are short range, $\sigma(\nu)$ must be a polynomial of ν. However the smallness of α_n obtained from this precise experiment reveals an embarrassing situation. Namely the n-Pb data in the MeV. region have the behavior characteristic of the long range force[2], and if we attribute it to the same electric polarization of the neutron, the value of α_n becomes around two orders of magnitude greater compared to what is obtained in the very low energy region.[3] Therefore we need another long range force which does not alter the coefficient of $\sqrt{\nu}$. An example of the required long range force is the strong Van der Waals force, because it gives rise to the singular term $\nu^{3/2}$ or $\nu^2 \log \nu$ in Eq.(1) as we shall see in the next section.

Since in the hadron physics, interactions are believed for a long time to be short range arising from the exchanges of a pion, a set of pions or heavier particles, the appearance of the strong Van der Waals force may sound strange. However in 1960's our view of hadrons changed from elementary particles to composite particles. In most of the composite model of hadron, the fundamental force is strong or super-strong Coulombic force, and it is responsible for the formation of the "neutral" bound states, and which are identified with the hadrons. Here "neutral" means total "charge" zero, in which "charge" corresponds to the fundamental Coulombic force. As in the case of the ordinary (electric) atoms, the quantum fluctuation gives rise to the Van der Waals interaction between the "neutral" composite particles, namely between hadrons. When the radius of the hadron and the "fine structure constant" $^*e^2$ are known, we can estimate the strength C of the Van der Waals potential , at least in the order of magnitude. It turns out that when the fundamental Coulombic force is super-strong such as in the case of the dyon model of Schwinger[4] , in which $^*e^2 = 137.04/4$, C becomes large enough to compete with the potential of the one-pion exchange. Therefore it is desirable to search for the strong Van der Waals interaction whenever sufficiently precise data are available. The neutron-Pb scattering has an advantage in the search, because for large r the strength C of the strong Van der Waals potential of the nuclear force is magnified by a factor A in the neutron-nucleus potential, where A is the mass number of the nucleus and we shall set $A = 208$ in this paper.

From the analysis of the S-wave phase shift data of the low energy proton-proton scattering, we have determined the parameters of the long range tail of nuclear potential $v(r) = -C/r^\alpha + \cdots$ in which $\alpha = 6.08$ and $C = 0.196$ in the unit of the Compton wave length of the neutral pion.[5] Once the tail of the nuclear potential is known, it is not difficult to compute the asymptotic behavior of the n-Pb potential and to determine

the singular behavior of the n-Pb scattering amplitude. Since the singular term has the form $(-t)^\gamma$, where $\gamma = (\alpha - 3)/2$, we can obseve the anomalous behavior of the n-Pb amplitude in two places. One is the anomalous behavior $(1-z)^\gamma$ of the amplitude at $z = 1$ for fixed ν, and the other is the anomolous term ν^γ in the partial wave amplitudes $a_\ell(\nu)$. The aim of the present paper is to predict the shapes and the magnitudes of these singular behaviors of the n-Pb amplitude by using the parameters of the long range force determined from the p-p scattering.

In section 2, the relations between the singular behavior of the amplitude at $t = 0$ and the asymptotic behavior of the potential at large r are described. In section 3, the anomaly of the angular distributions of the amplitudes are shown, in which the incident energies are fixed at $T_{lab} = 0.25$, 0.5, 0.75 and 1.0 MeV. respectively. In section 4, the once subtracted P-wave amplitude $a_1(\nu)/\nu$ is computed, and it is shown that it has a characteristic cusp $\nu^{\gamma-1}$ at $\nu = 0$. Section 5 is used for comments and discussions. In Appendix, the long range components of the nuclear potential obtained from the S-wave amplitude of the p-p scattering are summarized along with the brief explanation of the analysis of the p-p data.

2 Asymptotic form of the potential and the singular behavior of the amplitude

When we want to confirm the existence of the long range force unambiguously, it is desirable to use the difference of the analytical structure of the scattering amplitude $A(s,t)$. This is because the amplitude $A(s,t)$ is regular in the neighborhood of $t = 0$ if all the forces are short range, on the other hand when the long range force is acting an extra singularity appears at $t = 0$. It is fortunate that $t = 0$ is the end point of the physical region $-4\nu \le t \le 0$, and we can in principle settle the ploblem whether the strong interaction involves the long range force, when the sufficiently accurate data are given.

Since the potential $v(r)$ and the spectral function $A_t(s,t)$ of the scattering amplitude are connected by

$$v(r) = -\frac{1}{\pi m^2}\frac{1}{r}\int_0^\infty dt A_t(s,t) e^{-r\sqrt{t}} \quad , \tag{2}$$

the parameters of the threshold behavior of $A_t(s,t)$ and those of the asymptotic behavior of the potential, which are defined by

$$A_t(s,t) = \pi C' t^\gamma + \cdots \qquad and \qquad v(r) \sim -C\frac{1}{r^\alpha} + \cdots \tag{3}$$

respectively, relate to each other by

$$\alpha = 2\gamma + 3 \qquad and \qquad C = -\frac{2C'}{m^2}\Gamma(2\gamma+2) \quad . \tag{4}$$

From the threshold behavior of $A_t(s,t)$, the singular behavior of the amplitude $A(s,t)$ at $t = 0$ is determined :

$$A(s,t) = -\frac{\pi}{\sin\pi\gamma}C'(-t)^\gamma + \text{ (polynomial of } t \text{) } . \tag{5}$$

In particular when γ is an integer n, we must take the limit $\gamma \to n$ of Eq.(5), and the singular term becomes $(-1)^{n+1}C'(-t)^n \log(-t)$. For the case of the strong Van der Waals force, the power α of the asymptotic behavior of the potentials are $\alpha = 6$ and $\alpha = 7$ for the London type and the Casimir-Polder type respectively, and which correspond to $\gamma = 1.5$ and $\gamma = 2$ respectively.

Merits to use the large nucleus as the target of the neutron scattering are twofold. When we construct the neutron-nucleus potential by making the convolution of the nuclear potential and the form factor of the nucleus, the strength of the long range tail of the potential becomes A times large compared to that of the nuclear potential between nucleons, where A is the mass number of the nucleus and in our case $A = 208$. On the other hand, since the one-pion exchange term of the nuclear potential involves the factor $(\vec{\sigma}_n \cdot \vec{\sigma}_j)$ or $(\vec{\sigma}_n \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r})$, sum of the contributions from all the constituent nucleons cancels out. Therefore we may expect to get very clear view of the singularity of the long range force at $t = 0$, because the spectrum of the two-pion exchange starts at $t = 4$ and gives rise to an almost constant back ground in the small neighborhood of $t = 0$. Throughout of this paper, we shall use the neutral pion mass and its Compton wave length as the units of the energy and the length respectively. However because of the large radius r_1 of the nucleus, eg. for Pb^{208} r_1 is 4.42, we cannot neglect the back ground ploynomial function of t, when the domain of t becomes wide and lies out side of $\sqrt{-t} < 1/r_1$. In order to know the necessary degree of the back ground polynomials, in section 3 and 4 we shall also compute the n-Pb amplitude arising from short range potential, the potential of the σ-meson exchange in particular.

Since $t = -2\nu(1-z)$, the singular term $A^{sing}(s,t)$ becomes

$$A^{sing}(s,t) = -\frac{\pi}{\sin \pi\gamma} C'(2\nu)^{\gamma}(1-z)^{\gamma} \quad , \tag{6}$$

and for fixed ν, we must observe the singular behavior $(1-z)^{\gamma}$ in the neighborhood of $z = 1$. Another interesting property of the long range force is the anomaly of the threshold behavior of the partial wave amplitude $a_{\ell}(\nu)$. For the short range force, it is well-known the amplitude $a_{\ell}(\nu)$ is proportional to ν^{ℓ} at the threshold. This is because in the partial wave projection of the polynomial function of t, z^{ℓ} appears first in the term of t^{ℓ}, thus the factor ν^{γ} always appears in $a_{\ell}(\nu)$. On the other hand, the result of the partial wave projection of the singular term is

$$a_{\ell}^{sing}(\nu) = \frac{1}{2}\int_{-1}^{1} dz P_{\ell}(z) A^{sing}(s,t) = -\frac{\pi}{\sin \pi\gamma} C'(2\nu)^{\gamma} I_{\ell}(\gamma) \quad , \tag{7}$$

where the patial wave projection of $(1-z)^{\gamma}$ is

$$I_{\ell}(\gamma) = 2^{\gamma} \frac{(-\gamma)(1-\gamma)\cdots(\ell-1-\gamma)}{(1+\gamma)(2+\gamma)\cdots(\ell+1+\gamma)} \tag{8}$$

for $\ell > 0$ and for $\ell = 0$, $I_0(\gamma) = 2^{\gamma}/(1+\gamma)$. Therefore all the partial wave amplitudes have a term whose threshold behavior is proportional to ν^{γ}, if the long range force is acting. For example for the case of the Van der Waals force of the London type ($\gamma = 1.5$), all the partial waves other than S and P waves have the threshold behavior

$\nu^{1.5}$. Moreover although the threshold behavior of the P-wave is normal and we can consider $a_1(\nu)/\nu$, it involves a term proportional to $\sqrt{\nu}$ arising from the singular term $A^{sing}(s,t)$. Since Eqs.(7) and (8) indicate that the term of $\sqrt{\nu}$ has the negative sign, when the asymptotic force is attractive ($C' > 0$), the cusp of $a_1(\nu)/\nu$ at $\nu = 0$ must points upward. The shape of the P-wave in the low energy region is one of the ideal place to observe the effect of the long range force, because for the higher partial waves, we cannot obtain precise data in the low energy region. On the other hand, although the cusp of $\sqrt{\nu}$ with opposite sign appears in the once subtracted S-wave amplitude $(a_0(\nu) - a_0(0))/\nu$, extremely precise value of the scattering length $-a_0(0)$ is necessary to observe such a cusp. Therefore in the search of the long range force, we shall concentrate on the singular behavior of the angular distribution in the forward region $z = 1$ and on the shape of $a_1(\nu)/\nu$ of the P-wave in the low energy region.

3 Anomaly of the angular distribution of the neutron-Pb amplitude

Since the parameters of the long range component of the nuclear force are already known by the analysis of the S-wave amplitude of the proton-proton scattering, we can determine the potential of the neutron-nucleus scattering, at least in the region of the long range tail. It must be emphasized that the singular behavior of the amplitude at $t = 0$ is determined solely by the asymptotic behavior of the long range potential, and therefore the change of the potential for finite r, for example inside of the nucleus, does not have any effect on the singularity at $t = 0$. All the information on the long range component of the nuclear force is contained in the spectral function

$$A_t^{extra}(s,t) = \pi C' t^{\gamma} e^{-\beta t} \tag{9}$$

with

$$\gamma = 1.54 \quad , \quad C' = 0.175 \quad and \quad \beta = 0.0626 \tag{10}$$

in the unit of the neutral pion mass. The value of γ is close to that of the Van der Waals force of the London type($\gamma = 1.5$) and the sign of C' indicates that the force is attractive. In this section, we shall compute the amplitude of the neutron-Pb^{208} scattering using the parameters γ and C', and predict the shape of the singular behavior at $t = 0$. Such a prediction will be helpful in designing the neutron-Pb^{208} experiments of high precision and the data obtaind in the experiments will in turn improve the values of the parameters γ and C'.

In computing the convolution of the nuclear potential and the form factor $\rho(r)$ of the nucleus, for reason of simplicity, we shall choose $\rho(r)$ as the box form of radius r_1, namely $\rho(r)$ is zero and non-zero constant in $r > r_1$ and in $r < r_1$ respectively. This is permissible because we are interested in the singularity and it is not affected by the change of the potential at finite r. Among three parameters appeared in the spectral function $A_t^{extra}(s,t)$ of Eq.(9), β does not relate to the singularity, but it controls the depth of the n-Pb potential. Contrary to γ and C', β will be regarded as

a free parameter and whose value will be fixed by fitting to the scattering length of the S-wave amplitude of the n-Pb scattering. There is another free parameter r_1, size of the nucleus. We shall consider two amplitudes, in which $r_1 = 4.4$ and $r_1 = 3.5$ respectively. The former is the standard size of the nucleus of $A = 208$, however $r_1 = 4.4$ gives the slope of $\sqrt{\nu}\cot\delta_0(\nu)$ around 10 % higher. If we move r_1 to 3.5 it gives right value $d(\sqrt{\nu}\cot\delta_0(\nu))/d\nu = 1.9$, which was obtained in the transmission experiment of the n-Pb^{208} scattering. In the following, we shall designate these two amplitudes starting from the strong Van der Waals potential [vdw44] and [vdw35] respectively. In order to compare the cases of the long range forces with that of the short range force, the third amplitude of n-Pb will be considered starting from the nuclear potential of the sigma meson exchange with $m_\sigma = 4$. In the calculation we shall choose $r_1 = 4.4$ and the coupling constant g_σ^2 is fixed by fitting to the scattering length of the S-wave. This amplitude will be designated [sigma]. In figure 1, the potentials $V(r)$ of the n-Pb scatterings are shown.

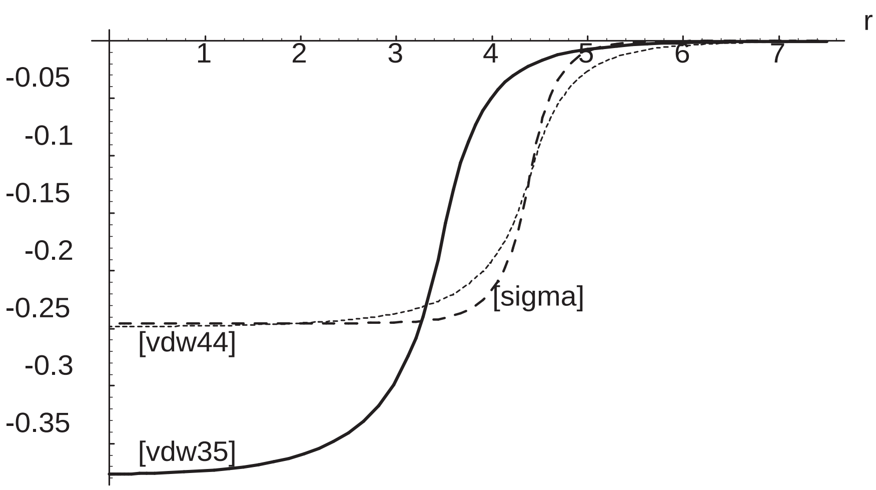

Figure 1: The potentials of neutron-Pb interactions [vdw35], [vdw44] and [sigma] are plotted against r. [vdw35] and [vdw44] are constructed from the nuclear potential of the strong Van der Waals type, whose radius of the nucleus r_1 is 3.5 and 4.4 respectively. Whereas [sigma] is constructed from the nuclear potential of σ-meson exchange, and r_1 =4.4 .

In this paper, we shall consider the n-Pb scattering in $T_{lab} \leq 1$ MeV.. Since ν and T_{lab} is proportional and $T_{lab} = 1$ MeV. corresponds to $\nu = 0.102123$, the domain of $-t$ necessary to make the patial wave projection is $0 \leq -t < 0.41$, whereas the nearest singularity on the t-plane arising from the exchange of the sigma meson is $t = 16$. Therefore we expect that the partial wave amplitudes $a_\ell(\nu)$ of [sigma] decreases very rapidly with ℓ. On the other hand, for the case of the Van der Waals interaction in which all the threshold behavior is proportional to ν^γ for $\ell \geq 2$, $a_\ell(\nu)$ does not decrease so rapidly even if ν is small. However we can see that the difference of the partial wave amplitudes from those of the Born term, namely $(a_\ell(\nu) - a_\ell^{(Born)}(\nu))$, decreases very

rapidly with ℓ for small ν. Therefore the amplitudes of [vdw44] and [vdw35] are

$$F(\nu,t) = F^{Born}(\nu,t) + \sum_{\ell=0}^{L}(2\ell+1)(a_\ell(\nu) - a_\ell^{(Born)}(\nu))P_\ell(z) \quad , \tag{11}$$

and we shall set $L=4$ because $|a_5(\nu) - a_5^{(Born)}(\nu)| < 10^{-4}$. Since the n-Pb potential $V(R)$ is the convolution of $\rho(r)$ and $v(r)$, the Born term of the n-Pb scattering amplitude $F^{Born}(\nu,t)$ is the product of the Fourier transformations of these functions.

The first one is

$$A\hat{\rho}(t) \equiv \int d^3 r e^{-i\vec{q}\cdot\vec{r}}\rho(r) = A\frac{3(\sin x - x\cos x)}{x^3}|_{x=qr_1} \qquad , \tag{12}$$

where q is the momentum transfer and relates to t by $t=-q^2$. Since $\hat{\rho}(t)$ is an even function of q, it is a regular function of t, and it is normalized as $\hat{\rho}(0)=1$. The second one is the Born term of the neutron-nucleon scattering:

$$A^{extra}(t) \equiv \frac{1}{\pi}\int dt' \frac{A_t^{extra}(s,t')}{t'-t} = \Gamma(\gamma+1)C'(-t)^\gamma e^{-\beta t}\Gamma(-\gamma,-\beta t,\infty) \quad , \tag{13}$$

in which the incomplete gamma function is used and whose definition is

$$\Gamma(g,a,b) = \int_a^b dx x^{g-1}e^{-x} \tag{14}$$

$\Gamma(g,p,\infty)$ can be divided into two terms: $(\Gamma(g) - \Gamma(g,0,p))$. If we use the separation, $A^{extra}(t)$ is written as the sum of the singular and the regular terms of t :

$$A^{extra}(t) = -\frac{\pi C'}{\sin\pi\gamma}(-t)^\gamma e^{-\beta t} - \Gamma(\gamma+1)\frac{C'}{\beta^\gamma}(-\beta t)^\gamma e^{-\beta t}\Gamma(-\gamma,0,-\beta t) \quad . \tag{15}$$

From Legendre's formula

$$p^{-g}e^p\Gamma(g,0,p) = \sum_{n=0}^{\infty}\frac{p^n}{g(g+1)(g+2)\cdots(g+n)} \quad , \tag{16}$$

we see that the second term of the r.h.s. of Eq.(15) is a regular function of $-\beta t$. By multiplying $A\hat{\rho}(t)$, we finally obtain the Born term of the scattering amplitude $F^{Born}(\nu,t)$

$$F^{Born}(\nu,t) = F_s(\nu,t)(-t)^\gamma + F_r(\nu,t) \quad , \tag{17}$$

where $F_s(\nu,t)$ and $F_r(\nu,t)$ are regular functions of t, and are defind by

$$F_s(\nu,t) = -\frac{\pi}{m\sin\pi\gamma}AC'e^{-\beta t}\hat{\rho}(t) \quad , \tag{18}$$

$$F_r(\nu,t) = -\frac{AC'}{m\beta^\gamma}\Gamma(\gamma+1)\{(-\beta t)^\gamma e^{-\beta t}\Gamma(-\gamma,0,-\beta t)\}\hat{\rho}(t) \tag{19}$$

respectively.

Contrary to the case of the short range force, the amplitudes with the long range force cannot be fitted by a few terms of the polynomials of z. In fact curves [vdw35] and [sigma] in the figures show such property, where [vdw35] and [sigma] are the scattering amplitudes $F(\nu,t)$ minus their S, P and D waves for the Van der Waals force and for the σ-meson exchange force respectively. Smallness of [sigma] means that the amplitude arising from the σ-meson exchange is fitted well by the quadratic function of z. In order to confirm that the singularity in [vdw35] is actually $F_s(\nu,t)(-t)^\gamma$, we must show the smallness of $\{F(\nu,t) - F_s(\nu,t)(-t)^\gamma\}$ minus the S, P and D waves. In the figure, the dotted curve is such a regular part of [vdw35]. Figure 2 and 3 are the graphs of $T_{lab} = 0.25$ and 0.5MeV. respectively, in which the full line is the [vdw35], the dashed line is the [sigma] multiplied by 10 and the dotted line is the regular part of [vdw35] multiplied by factor 100.

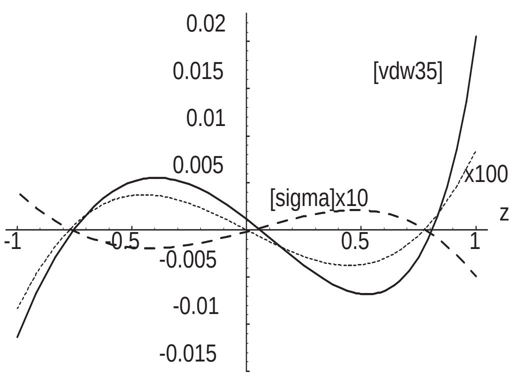

Figure 2: T_{lab} =0.25MeV.. Amplitudes $F(\nu,t)$ minus the S, P and D waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by 100.

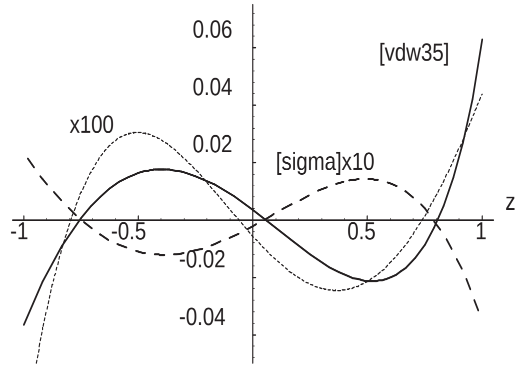

Figure 3: T_{lab} =0.50MeV.. Amplitudes $F(\nu,t)$ minus the S, P and D waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by 100.

Figure 4 and 5 are the graphs of $T_{lab} = 0.75$ and 1.0MeV. respectively, in which the contents are the same as fig.2 and fig.3 except the multiplicative factor, namely in these figures only the dotted curves are multiplied by factor 10. As the incident energy increases, even the amplitude arising from the short range potential becomes more and more difficult to be fitted by the quadratic function of z. Therefore we must try the fit by the cubic function for $T_{lab} = 0.75$ and 1.0MeV.. Figure 6 and 7 are the graphs of $T_{lab} = 0.75$ and 1.0MeV. respectively, in which the contents are the same as fig.4 and fig.5 except the subtracted functions are the cubic function, namely the S, P, D and F waves, rather than the quadratic function of z. And as in fig.4 and fig.5, only the regular part of [vdw35] (dotted curve) is multiplied by factor 10. When we draw curves, values of parameters of [vdw35] are

$$\gamma = 1.54 \quad , \quad \beta = 0.2517 \quad , \quad C' = 0.175 \quad and \quad r_1 = 3.5 \tag{20}$$

, whereas the nuclear potential of the σ-meson exchange is chosen as $v(r) = -0.59e^{-4r}/r$ and $r_1 = 4.4$ for [sigma].

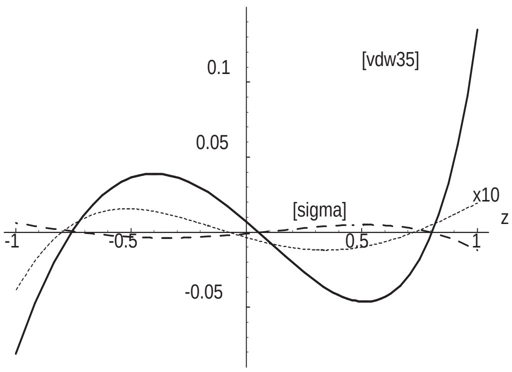

Figure 4: T_{lab} =0.75MeV.. Amplitudes $F(\nu, t)$ minus the S, P and D waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by factor 10.

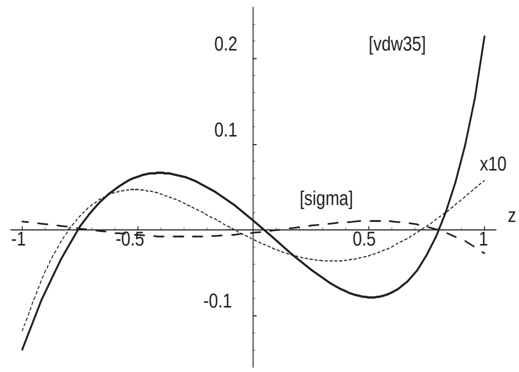

Figure 5: T_{lab} =1.0MeV.. Amplitudes $F(\nu, t)$ minus the S, P and D waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by factor 10.

When the precise n-Pb experimants are carried out, we shall obtain similar curves as [vdw35]. The content of our prediction is that the singular behaviors of the experimental curves will be removed by subtracting $F_s(\nu, t)(-t)^{\gamma}$ with the parameters given in Eq.(20), which are obtained from the data of the low energy p-p scattering. If the sufficiently accurate n-Pb data are available, we may expect to obtain the improved values of the long range parameters γ and C'.

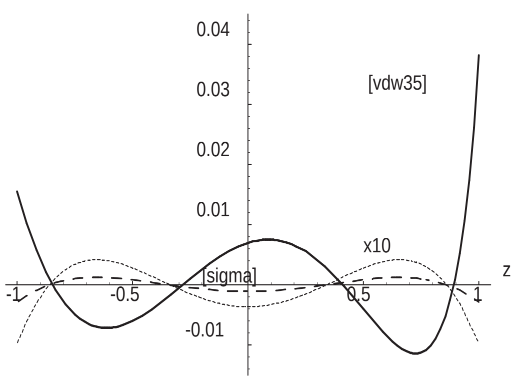

Figure 6: T_{lab} =0.75MeV.. Amplitudes $F(\nu, t)$ minus the S, P,D and F waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by factor 10.

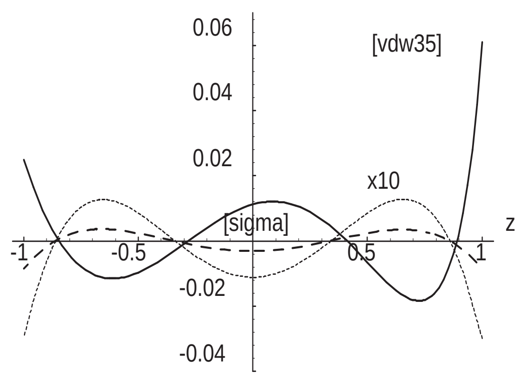

Figure 7: T_{lab} =1.0MeV.. Amplitudes $F(\nu, t)$ minus the S, P, D and F waves are plotted against z. The dotted curve, which is the regular part of [vdw35], is multiplied by factor 10.

4 The cusp of $a_1(\nu)/\nu$ at $\nu = 0$

Since the singular term $(-t)^\gamma$ is equal to $2^\gamma \nu^\gamma (1-z)^\gamma$, the singular behaviors appear in two places. One is the singular behavior $(1-z)^\gamma$ of the angular distribution at $z = 1$ for fixed ν, and the other is the singular term of ν^γ in the partial wave amplitude. In the previous section we studied the former, and the latter will be studied in this section. When $\gamma = 1.54$, ν^γ is the leading threshold behavior of the D and the higher partial waves rather than the standard threshold behavior ν^ℓ. For the P-wave $a_1(\nu)$, the threshold behavior is proportional to ν, and therefore we can use the once subtracted form $a_1(\nu)/\nu$. In $a_1(\nu)/\nu$ the singular term becomes $\nu^{\gamma-1}$, and which is a cusp at $\nu = 0$ because its slope is infinity. When the long range tail of the nuclear potential is attractive, from Eqs.(7) and (8) the coefficient of $\nu^{\gamma-1}$ is negative and therefore the cusp must point upward. On the other hand, the curve of [sigma] must be regular at $\nu = 0$. In figure 8, such curves of [vdw35], [vdw44] and [sigma] are shown. In order to make the display compact, since we do not have interest in the back ground regular functions, we subtract linear functions, which are chosen in such ways that curves come to ν-axis as close as possible in $0.02 < \nu < 0.03$.

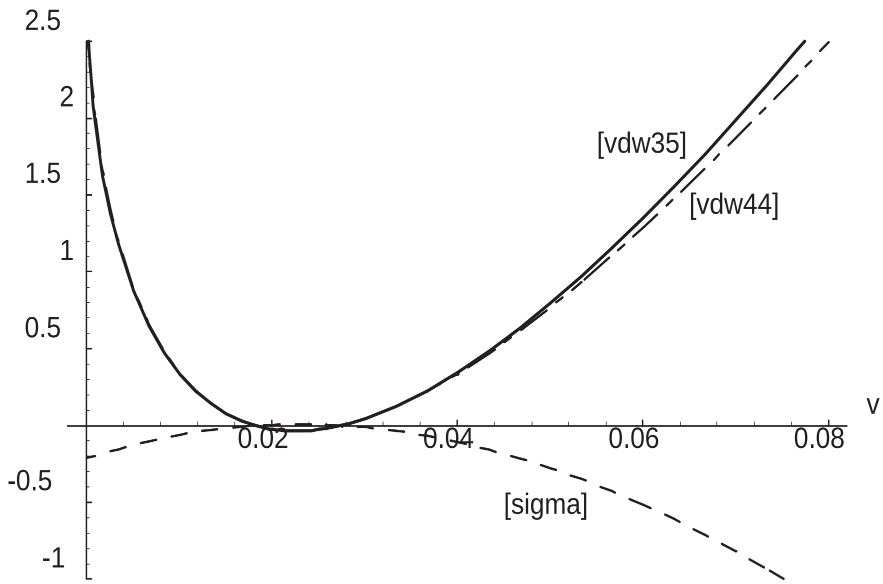

Figure 8: The Kantor amplitudes of the P-wave $K_1(\nu)/\nu$ are plotted against ν. Curves [vdw35] and [vdw44] arising from the strong Van der Waals force have cusps at $\nu = 0$, which point upward. On the other hand, the curve [sigma] arising from short range force is regular at $\nu = 0$.

If we speak precisely, the graphs in fig.8 are not the real parts of $a_1(\nu)/\nu$ but the once subtracted Kantor amplitude defined by[6]

$$\frac{K_1(\nu)}{\nu} = \mathrm{Re}\frac{a_1(\nu)}{\nu} - \frac{P}{\pi}\int_0^\infty d\nu' \frac{\mathrm{Im}a_1(\nu')}{\nu'(\nu'-\nu)} \quad , \tag{21}$$

where P stands for Cauchy's pricipal value integration. It is important that the unitarity cut in $0 \le \nu < \infty$ of $a_1(\nu)/\nu$ is removed in $K_1(\nu)/\nu$. Therefore $K_1(\nu)/\nu$ has a wide domain of analyticity, and which is necessary to observe the delicate difference of behaviors arising from the spectrum on the left hand cut $-\infty < \nu \le m_1^2/4$ of the short range force, where m_1 is the lightest mass exchanged, and from the spectrum of the long range force in $-\infty < \nu \le 0$. It is fortunate that, for the P and higher partial waves, the integrations of Eq.(21) are small and in their estimation, very precise phase shift data are not required. This is because the threshold behavior of $\text{Im}a_1(\nu')/\nu'$ is $\nu^{3/2}$ even for the P-wave. On the other hand for the S-wave, the threshold behavior of $\text{Im}a_0(\nu')/\nu'$ is $1/\sqrt{\nu}$ and the estimation of the principal value integration requires very precise data of the phase shift $\delta_0(\nu)$. Another difficulty to use the S-wave amplitude is that we need to know extremely precise value of the scaatering length $-a_0(0)$, since in cmputing the once subtracted Kantor amplitude, we must know $(a_0(\nu) - a_0(0))/\nu$. Therefore the S-wave is not the suitable place to observe the singularity $\nu^{\gamma-1}$, unless we can overcome these difficulties. If we consider that we do not have sufficient data of the D and higher partial waves in the low energy region, the P wave is the most suitable place to observe the singularity ν^{γ} of the partial waves.

5 Remarks and Comments

Since the Van der Waals force is universal[7] , its existence in the nucleon-nucleon scattering implies the Van der Waals interaction also in other processes such as in the pion-pion[8] and in the pion-nucleon scatterings. The possibility to find such an extra force depends on the precision of the data and the possibility to prepare the wide domain of analyticity. The low energy data of the p-p scattering is prominent in their accuracy, whereas the pion-pion process is prominent in its possibility to prepare the wide analytic domain, because the two-pion exchange spectrum can be constructed from the pi-pi amplitudes themselvs. In the Appendix, we see a cusp at $\nu = 0$ in the once subtracted Kantor amplitude of the S-wave of the p-p scattering, in which the one-pion exchange spectrum is removed. Similarly the characteristic cusp at $\nu = 0$ is observed in $a_1(\nu)/\nu$ of the pion-pion scattering, in which the unitarity cut and the cut of the two-pion exchange are removed.[8]

The third place easy to observe the strong Van der Waals force is the low energy neutron-nucleus amplitude, because the strength C of the strong Van der Waals potential of the nuclear force is magnified by factor A, the mass number. Although the effect of the Van der Waals force decreases as ν^{γ} for small ν, in the neuton-nucleus scattering the large value of the strength AC allows us to observe it even at the smaller energy. It is advantageous to use the lower energy data in the determinations of the paremeters γ and C' of the threshold behavior appeared in the extra spectrum $A_t^{extra}(s,t) = \pi C' t^{\gamma} e^{-\beta t}$, because they are determined without being disturbed by the parameter β, which specifies the deviation from the threshold behavior.

Therefore our proposal in this paper is to measure the angular distribution of the cross section of the neutron-nucleus scattering such as n-Pb^{208} precisely for fixed energy

around T_{lab} =1MeV., and to determine the parameters of the asymptotic tail of the long range component of the nuclear potential. Although the values of such parameters are known from the low energy proton-proton data, the independent determination will serve to confirm the actual existence of the strong Van der Waals interaction in the nuclear force.

Appendix

When we extract the information of the long range force from the partial wave amplitude $h_\ell(\nu)$, first of all we must remove the unitarity cut, and then the known near-by singularities. In this way we can prepare a function with the wide domain of analyticity in the neighborhood of $\nu = 0$, and which is helpful to observe the extra singularity at $\nu = 0$ arising from the long range interaction, if it exists. Since the phase shift data of the S-wave of the low energy proton-proton scattering is most accurate in the hadron physics, we shall consider the once subtracted S-wave amplitude $(h_0(\nu) - h_0(0))/\nu$. If we make the Kantor amplitude by

$$K_0^{once}(\nu) = \mathrm{Re}(h_0(\nu) - h_0(0))/\nu - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}h_0(\nu')}{\nu'(\nu'-\nu)}d\nu' \tag{22}$$

, the unitarity cut is removed. Since the one-pion exchange (OPE) contribution is

$$h_0^{1\pi}(\nu) = \frac{1}{4}\frac{g^2}{4\pi}\frac{1}{4\nu}\log(1+4\nu) \tag{23}$$

, which has a left hand cut starting from $\nu = -1/4$, the OPE cut in the Kantor amplitude is removed in

$$\tilde{K}_0^{once}(\nu) = K_0^{once}(\nu) - \frac{(h_0^{1\pi}(\nu) - h_0^{1\pi}(0))}{\nu} \quad . \tag{24}$$

When we have precise phase shift data, we can evaluate $-\tilde{K}_0^{once}(\nu)$, and it is shown in figure 9 and 10. Although the spectrum of the two-pion exchange starts at $\nu = -1$, in the threshold region the spectrum is very small, because it arises from the partial wave projection of the continuous spectrum $A_t^{2\pi}(s,t)$. If we replace $A_t^{2\pi}(s,t)$ by the delta function of the σ-meson exchange, the left hand cut of $-\tilde{K}_0^{once}(\nu)$ starts at $\nu = -4$. Therefore if the nuclear force is short range due to the exchanges of a set of pions, $-\tilde{K}_0^{once}(\nu)$ must be almost constant with very small slope and extremely small curvature in the neighborhood of $\nu = 0$.

However figures 9 and 10 indicate this is not the case, but there is a cusp at $\nu = 0$ (closed circles). This cusp is fitted by an extra spectrum of the long range interaction of the form

$$A_t^{extra}(4m^2, t) = \pi C' t^\gamma e^{-\beta t} \quad . \tag{25}$$

More explicitely, it is fitted by

$$\frac{h_0^{extra}(\nu) - h_0^{extra}(0)}{\nu} = 2C'\int_0^\infty dt\, t^\gamma e^{-\beta t}\{\frac{1}{2\nu}Q_0(1+\frac{t}{2\nu}) - \frac{1}{t}\}\frac{1}{\nu} \tag{26}$$

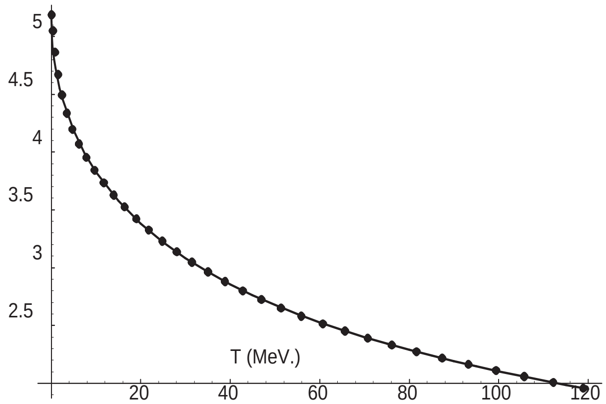

Figure 9: $-\tilde{K}_0^{once}(\nu)$ is plotted against T_{lab} (MeV.). The closed circles are evaluated from the S-wave phase shifts of the p-p scattering. The error bars are smaller than the size of the circles in T_{lab} >4MeV.. The curve is the results of the fit, in which the spectrum with three free parameters of Eq.(25) is searched.

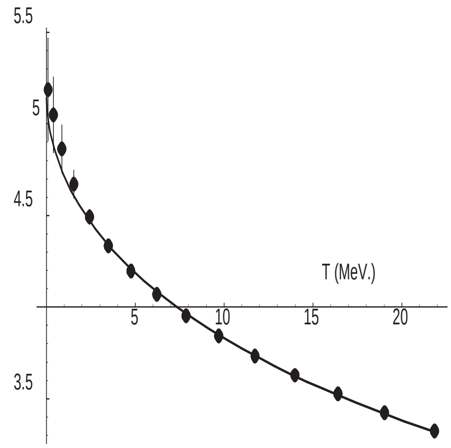

Figure 10: The same graph in the lower energy region as the figure 9 is shown. The error bars are drawn. The cusp at $\nu = 0$, which points upward, indicates the strong long range force with the attractive sign.

, in which the extra factor 2 in front of the integration appears because the contributions from the spectrum A_u as well as from A_t must be included. The integration of Eq.(26) can be written in terms of the generalized hypergeometric function ${}_pF_q(x)$ and of the confluent hypergeometric function $F(a,b,x)$.

$$\frac{h_0^{extra}(\nu) - h_0^{extra}(0)}{\nu} = 2(\nu^{\gamma-1}\xi_s(\nu) + \xi_r(\nu)) \tag{27}$$

, where $\xi_s(\nu)$ and $\xi_r(\nu)$ are regular functions of ν and are difined by

$$\xi_s(\nu) = -C'\frac{4^\gamma}{\gamma+1}\frac{\pi}{\sin\pi\gamma}F(1+\gamma, 2+\gamma, 4\beta\nu) \tag{28}$$

and

$$\begin{aligned}\xi_r(\nu) &= C'\frac{\Gamma(\gamma)}{\beta^\gamma\nu}({}_{\{1,1\}}F_{\{2,1-\gamma\}}(4\beta\nu) - 1) \\ &= C'\frac{\Gamma(\gamma)}{\beta^\gamma\nu}(\sum_{n=1}^{\infty}\frac{(4\beta\nu)^n}{(1-\gamma)\cdots(n-\gamma)}\frac{1}{n+1})\end{aligned} \tag{29}$$

respectively. Results of the chi-square fits are

$$\gamma = 1.543 \quad , \qquad \beta = 0.06264 \quad , \qquad C' = 0.1762 \qquad and \qquad \chi = 0.441 \tag{30}$$

in the unit of the neutral pion mass. Among three parameters, γ and C' are the threshold parameters of the long range force, whereas β is necessary to make the integration

convergent, and $\sqrt{\beta}$ must be the order of magnitude of the nucleon radius. In constructing the neutron-Pb potential, although we use the values of γ and C' given in Eq.(30), β is left as a free parameter and later fixed by fitting to the scattering length of the n-Pb amplitude. Up to this point we have not considerd the ordinary Coulomb interaction and the vacuum polarization. Therefore the procedure explained above is applicable only to the neutron-neutron data. When we include the electromagnetic interaction, some modifications are necessary in the constructions of the singularity free functions $K_0(\nu)$ from the phase shift functions. An explicit construction of the Kantor amplitude from the phase shift $\delta_0^E(\nu)$ are explained in other paper.[5]

Finally it will be helpful to write the normalization of the amplitudes explicitly:

$$A(s,t) = \sum_{\ell=0}^{\infty}(2\ell+1)h_\ell(\nu)P_\ell(z) \quad with \quad h_\ell(\nu) = \frac{\sqrt{m^2+\nu}}{\sqrt{\nu}}e^{i\delta_\ell(\nu)}\sin\delta_\ell(\nu) \tag{31}$$

and

$$f(\nu,t) = \sum_{\ell=0}^{\infty}(2\ell+1)a_\ell(\nu)P_\ell(z) \quad with \quad a_\ell(\nu) = \frac{1}{\sqrt{\nu}}e^{i\delta_\ell(\nu)}\sin\delta_\ell(\nu) \quad . \tag{32}$$

Therefore $f(\nu,t) = A(s,t)/m$ in the approximation $\nu \ll m^2$.

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